

UNI 2012-2

En la siguiente ecuación trigonométrica

$$\cos^4\left(\frac{x}{2}\right) - \frac{1}{8}\cos(2x) = \frac{7}{8}$$

El número de soluciones en $[0; 2\pi]$ es:

- A) 1
- B) 2
- C) 3
- D) 4
- E) 5

Resolución

$$2\left(2\cos^2\frac{x}{2}\right)^2 - \cos 2x = 7$$

$$2(1 + 2\cos x + \cos^2 x) - \cos 2x = 7$$

$$2 + 4\cos x + 2\cos^2 x - (2\cos^2 x - 1) = 7$$

$$\cos x = 1$$

$$\rightarrow x = 0; 2\pi$$

UNI 2015-1

Determine para qué valores de $x \in \langle 0; 2\pi \rangle$ se cumple:

$$\frac{\cot^2(x) + 4}{2\sin^2(x) + 5\sin(x) - 3} > 0$$

A) $\left\langle \frac{\pi}{6}; \frac{\pi}{2} \right\rangle$

B) $\left\langle \frac{\pi}{6}; \frac{3\pi}{4} \right\rangle$

C) $\left\langle \frac{\pi}{6}; \frac{5\pi}{6} \right\rangle$

Resolución

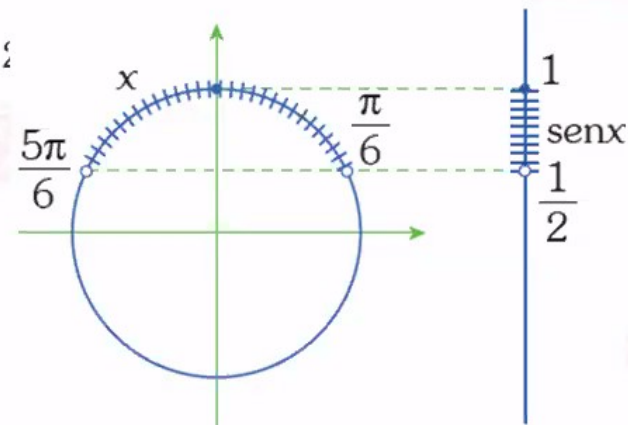
Factorizamos

$$\frac{\cot^2 x + 4}{(\sin x + 3)(2\sin x - 1)} > 0; \quad x \in \langle 0; 2\pi \rangle$$

Como $\cot^2 x + 4 \geq 4$ y $\neq 0$

$$\rightarrow 2\sin x - 1 > 0$$

$$\rightarrow \frac{1}{2} < \sin x \leq 1$$



$$\therefore x \in \left\langle \frac{\pi}{6}; \frac{5\pi}{6} \right\rangle$$

UNI 2016-1

Al resolver la ecuación

$\text{sen}(2x) - 12(\text{sen}(x) - \cos(x)) + 12 = 0$,
obtenemos como soluciones:

- A) $k\pi; k \in \mathbb{Z}$
- B) $2k\pi$ y $\left(k + \frac{1}{2}\right)\pi; k \in \mathbb{Z}$
- C) $2k\pi$ y $k\pi; k \in \mathbb{Z}$
- D) $(2k+1)\pi$ y $\left(2k + \frac{1}{2}\right)\pi; k \in \mathbb{Z}$
- E) $(3k+1)\pi$ y $\left(k + \frac{1}{2}\right)\pi; k \in \mathbb{Z}$

Resolución

De la condición

$$\text{sen}2x - 12(\text{sen}x - \cos x) + 12 = 0$$

$$1 - \text{sen}2x + 12(\text{sen}x - \cos x) - 13 = 0$$

$$(\text{sen}x - \cos x)^2 + 12(\text{sen}x - \cos x) - 13 = 0$$

$$\begin{array}{l} \text{sen}x - \cos x \quad \quad \quad 13 \\ \text{sen}x - \cos x \quad \quad \quad -1 \end{array}$$

$$(\text{sen}x - \cos x + 13)(\text{sen}x - \cos x - 1) = 0 \quad \rightarrow \quad \text{sen}x - \cos x - 1 = 0$$

$$\sqrt{2} \text{sen}\left(x - \frac{\pi}{4}\right) = 1$$

Luego

$$x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{4};$$

$$x - \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4};$$

$$x = 2k\pi + \frac{\pi}{2}$$

$$x = 2k\pi + \pi; k \in \mathbb{Z}$$

resuelva

$$(\operatorname{sen} x - \cos x + 1)(\operatorname{sen} x + \cos x - 1) > 0$$

para $\langle 0; 2\pi \rangle$

A) $\left\langle 0; \frac{\pi}{2} \right\rangle$

B) $\langle 0; \pi \rangle$

C) $\langle 0; 2\pi \rangle$

D) $\left\langle \frac{\pi}{2}; \frac{3\pi}{2} \right\rangle$

E) $\left\langle 0; \frac{\pi}{2} \right\rangle \cup \left\langle \frac{3\pi}{2}; 2\pi \right\rangle$

Resolución

$$(\operatorname{sen} x - \cos x + 1)(\operatorname{sen} x + \cos x - 1) > 0$$

$$\operatorname{sen}^2 x - (1 - \cos x)^2 > 0$$

$$1 - \cos^2 x - (1 - 2\cos x + \cos^2 x) > 0$$

$$2\cos x - 2\cos^2 x > 0$$

$$2\cos x(1 - \cos x) > 0 \dots (*)$$

$$0 \leq 1 - \cos x$$

$$\rightarrow \cos x > 0$$

$$\text{obtenemos: } 0 < \cos x < 1,$$

E

iVCam

① SOL

$$0 < x < 2\pi$$

$$\cos 4x + 2\sin 2x + 3 = 0$$

$$1 - 2\sin^2 2x + 2\sin 2x + 3 = 0$$

$$0 = \sin^2 2x - \sin 2x - 2$$

$$\sin 2x \quad -2$$

$$\sin 2x \quad +1$$

$$\sin 2x = -1$$

$$\sin 2x = 2$$



$$x = \frac{3\pi}{4} / \frac{7\pi}{4} / \frac{11\pi}{4}$$

iVCam

① SOL.

$$0 < x < 2\pi$$

$$\cos 4x + 2 \sin 2x + 3 = 0$$

$$1 + \cos 4x + 2 + 2 \sin 2x = 0$$

$$2 \cos^2 2x + 2(\sin x + \cos x)^2 = 0$$

$$\cos 2x = 0$$



$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\sin x = -\cos x$$

$$\tan x = -1$$



$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

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$\Sigma = \bigcirc$

② $0 \leq x < 2\pi$
SOMA DE SOL.

$$\frac{\tan x - \sqrt{3} \cot x + 1 - \sqrt{3}}{\tan 3x} = 0$$

$$3x \neq \frac{n\pi}{2} \rightarrow x \neq \frac{n\pi}{6}$$

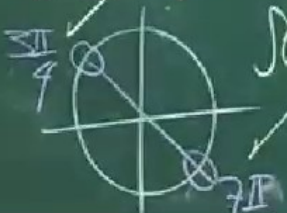
$$\tan x - \sqrt{3} \cot x + 1 - \sqrt{3} = 0$$

$$\tan x + 1 - \sqrt{3}(\cot x + 1) = 0$$

$$\tan x(\tan x + 1) - \sqrt{3}(1 + \tan x) = 0$$

$$(\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\tan x = -1 \quad \vee \quad \tan x = \sqrt{3}$$



$$\text{SOMA} = \frac{5\pi}{2}$$



③ Σ

$\Sigma = \bigcirc$

iVCam

$\Sigma = \bigcirc$

② $0 \leq x < 2\pi$
SOMA DE SOL.

$$\frac{\tan x - \sqrt{3} \cot x + 1 - \sqrt{3}}{\tan 3x} = 0$$

$$3x \neq \frac{n\pi}{2} \rightarrow x \neq \frac{n\pi}{6}$$

$$\tan x - \sqrt{3} \cot x + 1 - \sqrt{3} = 0$$

$$\tan x + 1 - \sqrt{3}(\cot x + 1) = 0$$

$$\tan x(\tan x + 1) - \sqrt{3}(1 + \tan x) = 0$$

$$(\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\tan x = -1 \quad \vee \quad \tan x = \sqrt{3}$$

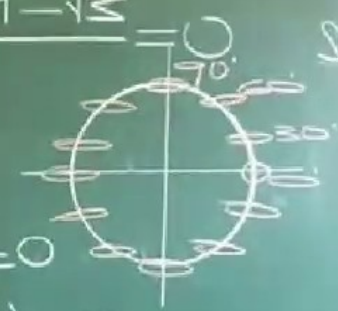


$$\text{SOMA} = \frac{5\pi}{2}$$



③

$\Sigma = \bigcirc$



PROBLEMA 6

Al resolver el sistema de ecuaciones

$$\operatorname{sen}^2 x = \operatorname{sen} y$$

$$\cos^4 x = \cos y$$

$(x; y)$ es la solución del sistema de las proposiciones.

I. $x = n\pi + \frac{\pi}{2}; y = 2m\pi + \frac{\pi}{2}; m; n \in \mathbb{Z}$

II. $x = n\pi; y = 2m\pi; m; n \in \mathbb{Z}$

III. $x = n\pi; y = 2m\pi + \frac{\pi}{2}; m; n \in \mathbb{Z}$

A) solo I

B) solo II

C) I y III

D) I y II

E) II y III

nota

$$\forall n \in \mathbb{Z}^+ : \operatorname{sen}^{2n} x + \cos^{2n} x \leq 1$$

Resolución



$$\operatorname{sen}^4 x + \cos^4 x \leq 1$$

primer caso :

segundo caso :

$$\operatorname{sen}^2 y + \cos y \leq 1$$

$$\cos y = 0 \dots \dots \text{I}$$

$$\cos y = 1 \dots \dots \text{I}$$

$$1 - \cos^2 y + \cos y \leq 1 \rightarrow \cos x = 0 \dots \dots \text{II}$$

$$\rightarrow \cos x = \pm 1 \dots \dots \text{II}$$

$$\cos y(1 - \cos y) \leq 0 \rightarrow \operatorname{sen}^2 x = 1 \dots \dots \text{III}$$

$$\operatorname{sen}^2 x = 0 \dots \dots \text{III}$$

$$\underbrace{\quad \quad}_{\geq 0} \quad \underbrace{\quad \quad}_{\geq 0}$$

$$\rightarrow \operatorname{sen} y = 1 \dots \dots \text{IV}$$

$$\operatorname{sen} y = 0 \dots \dots \text{IV}$$

de I y IV :

de I y IV

$$y = 2m\pi + \frac{\pi}{2}$$

$$y = 2m\pi$$

de II y III :

de II y III :

$$x = (2n + 1)\frac{\pi}{2}$$

$$x = n\pi$$

entonces :

$$\cos y(1 - \cos y) = 0$$

I, verdadero

II, verdadero

CLAVE: D

PROBLEMA 8

Resuelva la inecuación

$$\sin 2x + 3 \sin x > \cos 2x + 3 \cos x + 1$$

si, $0 < x < 2\pi$.

A) $\left\langle \frac{\pi}{4}; \frac{\pi}{2} \right]$

B) $\left\langle \frac{\pi}{2}; \frac{5\pi}{4} \right\rangle$

C) $\left\langle \frac{\pi}{4}; \frac{5\pi}{4} \right\rangle$

D) $\left\langle \frac{\pi}{4}; \frac{3\pi}{4} \right\rangle$

E) $\left\langle \frac{\pi}{4}; \pi \right\rangle$

Resolución



$$\sin 2x + 3 \sin x > \cos 2x + 3 \cos x + 1$$

$$0 > 1 + \cos 2x - \sin 2x + 3 \cos x - 3 \sin x$$

$$0 > 2 \cos^2 x - 2 \sin x \cos x + 3(\cos x - \sin x)$$

$$0 > 2 \cos x (\cos x - \sin x) + 3(\cos x - \sin x)$$

$$0 > (\cos x - \sin x)(2 \cos x + 3)$$

$$\cos x - \sin x < 0$$

$$\frac{1}{\sqrt{2}}(\cos x - \sin x) < 0$$

$$\cos\left(\frac{\pi}{4} + x\right) < 0$$

$$\left(\frac{\pi}{4} + x\right) \in \left\langle \frac{\pi}{2}; \frac{3\pi}{2} \right\rangle$$

$$\therefore x \in \left\langle \frac{\pi}{4}; \frac{5\pi}{4} \right\rangle$$

CLAVE: C

PROBLEMA 9

Al resolver la inecuación trigonométrica

$$\frac{\operatorname{sen} 4x + \cos 2x}{\operatorname{sen} 2x + \cos x} > 0$$

pero $x \in \left\langle 0; \frac{3\pi}{4} \right\rangle$

se obtiene por conjunto solución $\langle a; b \rangle \cup \langle c; d \rangle$
calcule $a+b+c+d$.

- A) π B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{4}$
D) $\frac{4\pi}{3}$ E) $\frac{7\pi}{4}$

Resolución

$$\frac{\operatorname{sen} 4x + \cos 2x}{\operatorname{sen} 2x + \cos x} > 0$$

$$\frac{\cos 2x(2\operatorname{sen} 2x + 1)}{\cos x(2\operatorname{sen} x + 1)} > 0$$

$$2\operatorname{sen} x + 1 > 0$$

$$\frac{\cos 2x(2\operatorname{sen} 2x + 1)}{\cos x} > 0 \dots I$$

analizamos x , por intervalos

i) **si**, $x \in \left\langle 0; \frac{\pi}{2} \right\rangle$

$$\cos x > 0$$

$$2x \in \langle 0; \pi \rangle$$

$$2\operatorname{sen} 2x + 1 > 0$$

$$\frac{\cos 2x(2\operatorname{sen} 2x + 1)}{\cos x} > 0$$

entonces :

$$\cos 2x > 0$$

$$2x \in \left\langle 0; \frac{\pi}{2} \right\rangle$$

$$\therefore CS_1 = \left\langle 0; \frac{\pi}{4} \right\rangle$$

ii) **si**, $x \in \left\langle \frac{\pi}{2}; \frac{3\pi}{4} \right\rangle$

$$\cos x < 0$$

$$2x \in \left\langle \pi; \frac{3\pi}{2} \right\rangle$$



$$\cos 2x < 0$$

se cumplirá la condición **I**, si :

$$2\operatorname{sen} 2x + 1 > 0$$

$$0 > \operatorname{sen} 2x > \frac{-1}{2}$$

$$\operatorname{sen} \pi > \operatorname{sen} 2x > \operatorname{sen} \frac{7\pi}{6}$$

$$\pi < 2x < \frac{7\pi}{6}$$

$$\frac{\pi}{2} < x < \frac{7\pi}{12}$$

$$\therefore CS_2 = \left\langle \frac{\pi}{2}; \frac{7\pi}{12} \right\rangle$$

CLAVE:D

